



**PLAXIS**

**Ground response analysis in PLAXIS**

**2D**

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*Edited by:*

A. Laera  
*PLAXIS bv, The Netherlands*

R.B.J. Brinkgreve  
*Delft University of Technology & PLAXIS bv, The Netherlands*

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## 1 INTRODUCTION

Earthquakes propagate from the bedrock to the surface through the overlying soil deposit, which acts as a filter and modify the ground motion characteristics. The variation of the seismic waves in terms of amplitude, duration and frequency content at any depth can be evaluated through a ground response analysis. The local soil stratigraphy, the material properties, the site topography, the ground water table depth and the characteristics of the earthquake (for example, duration, peak acceleration, frequency content) have a high influence on the ground response and on the modelling strategy. The ground response analysis of a soil deposit can be considered as a necessary preliminary study for the dynamic analysis of a structure, since its seismic response is influenced by the geological and geotechnical properties of the supporting soil. Due to its filter effect, the soil deposit modifies the seismic waves by amplifying the signal at some specific frequencies and damping it at some others. If the frequency at which the maximum amplification of the ground motion occurs is close to the natural frequency of the overlying structure, the building and the ground motion are in resonance with one another. This means that the system oscillates with very high amplitudes that can cause great damages in the building.

The site response analysis is performed in free field conditions, i.e. the motion that occurs in the soil layers at the depth of interest (for instance, at the foundation level in the case of a building with shallow foundation) is determined by applying the selected earthquake at the bedrock, in the absence of any structure or excavation. In some cases, a 'deconvolution' analysis may be needed, in order to convert the seismic signal recorded at the outcrop rock site into the earthquake to be applied at the bedrock surface. The free field surface motion is quantified in terms of amplification ratio, frequency content, peak ground acceleration and spectral acceleration.

For a specific site, it is required a probabilistic study to determine all the possible seismic events that can occur. The output used for the dynamic analysis of a structure represents the average result of several ground response analyses performed for the same site by applying a certain number of different seismic input signals (generally not less than 3), properly selected from the earthquake database and scaled at the desired value of maximum acceleration.

This example (with elaborated background information) concerns the one-dimensional wave propagation analysis through a clay deposit 40 m thick above the bedrock. One-dimensional analysis can be performed when the soil layers and the bedrock surface are horizontal and they extend to infinity, and the seismic waves coincide with shear waves propagating vertically from the underlying bedrock. This last assumption can be justified considering that the seismic waves, propagating from the earthquake source through the soil, are bent by successive refractions into a nearly vertical path (according to Snell's law of refraction). The problem can be modelled through a soil column with specific features, as described in the following sections.

Objectives:

- Modelling non linear behaviour of the soil using the HS small model
- Modelling boundary conditions for one-dimensional wave propagation with tied degrees of freedom



## 2 GEOMETRY

The soil layer consists of a lithological homogeneous clay deposit 40 m thick above the bedrock. To create the geometry model, follow these steps:

### 2.1 GENERAL SETTINGS

- Start the *Input* program and select *Start a new project* from the *Quick select* dialog box.
- In the *Project* tabsheet of the *Project properties* window, enter an appropriate title.
- In the *Model* tabsheet keep the default options for *Model* and *Elements*.
- Keep the default values for units and the general parameters.
- Set the model dimensions to  $x_{min} = 0.0$  m,  $x_{max} = 1.5$  m,  $y_{min} = 0.0$  m,  $y_{max} = 40.0$  m and press *OK* to close the *Project properties* window (Figure 2.1).

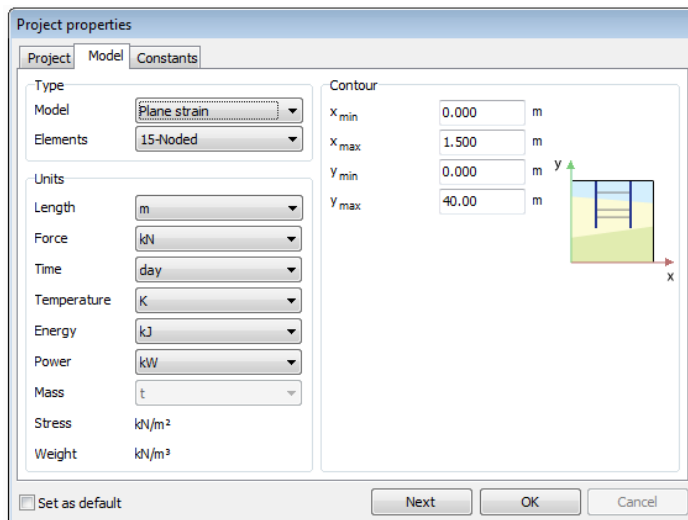


Figure 2.1 *Model tabsheet* of the *Project properties* window.

The horizontal dimension of the model is chosen in the order of the required element length, considering the applied seismic load and the characteristics of the soil that will be explained in the next sections.

The dynamic time that needs to be specified for the dynamic calculation phase is always expressed in seconds, even if the unit of time selected in the Project properties window is different. The specified unit is taken into account if other type of time dependent calculation phases are included in the project.

## 2.2 DEFINITION OF SOIL STRATIGRAPHY AND MATERIAL PROPERTIES

The soil deposit consists of a homogeneous clay layer. The water table is supposed to be sufficiently deep so that drained condition can be considered.

To define the soil stratigraphy:

- Create a borehole at  $x = 0$ . The *Modify soil layers* window pops up.
- Add one soil layer to the borehole. Locate the ground level at  $y = 40$  m by assigning 40 to the *Top* level. The *Bottom* is automatically set equal to 0.0 m. The layer stratigraphy is shown in Figure 2.2.

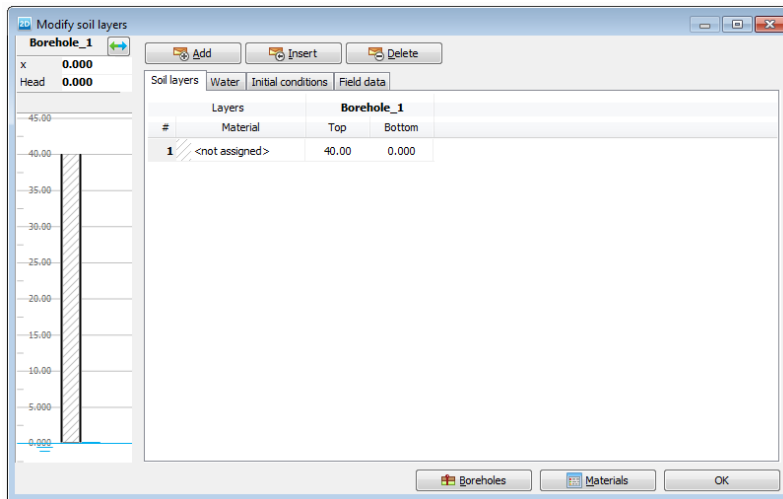


Figure 2.2 *Modify soil layers* window

The soil deposit consists of a clay layer modelled with the HS small model to account for hysteretic damping and stiffness increase with depth.

- Click on *Materials* to open the *Material sets* window.
- Create a data set under *Soil and interfaces* set type according to the information given in Table 2.1.
- Assign the material data set to the soil layer in the borehole.
- Close the *Modify soil layers* window and proceed to the *Structures* mode.

In general, the soil stiffness is characterized by non-linear behaviour, both stress and strain dependent, i.e., even in a homogeneous lithological soil deposit, the soil stiffness varies with depth and its value decays with the strain level induced by the loading. The maximum strain at which the soil behaviour can be considered purely elastic is very small (in the order of  $1 \cdot 10^{-6}$ ). The soil stiffness associated with this strain range is indicated as initial stiffness and its value decays by increasing the strain amplitude according to the characteristic S-curve in the logarithmic scale (Figure 2.3, modulus reduction curve).



Table 2.1 Material properties of the subsoil layer.

Parameter	Name	Value	Unit
General			
Material model	Model	HS small	-
Type of material behaviour	Type	Drained	-
Soil unit weight above phreatic level	$\gamma_{unsat}$	19	$kN/m^3$
Soil unit weight above phreatic level	$\gamma_{sat}$	19	$kN/m^3$
Rayleigh coefficient $\alpha$	Rayleigh $\alpha$	0.09558	-
Rayleigh coefficient $\beta$	Rayleigh $\beta$	0.7920E-3	-
Parameters			
Secant stiffness in standard drained triaxial test	$E_{50}^{ref}$	3250	$kN/m^2$
Tangent stiffness for primary oedometer loading	$E_{oed}^{ref}$	3250	$kN/m^2$
Unloading / reloading stiffness	$E_{ur}^{ref}$	9750	$kN/m^2$
Power for stress-level dependency of stiffness	$m$	0.8	-
Cohesion	$c'_{ref}$	10	$kN/m^2$
Friction angle	$\phi'$	24	degrees
Dilatancy angle	$\psi$	0	degrees
Shear strain at which $G_s = 0.722G_0$	$\gamma_{0.7}$	0.0007	-
Shear modulus at very small strains	$G_0^{ref}$	31200	$kN/m^2$
Poisson's ratio	$\nu'_{ur}$	0.2	-
Overconsolidation Ratio	OCR	1	-
Pre-overburden pressure	POP	0	-

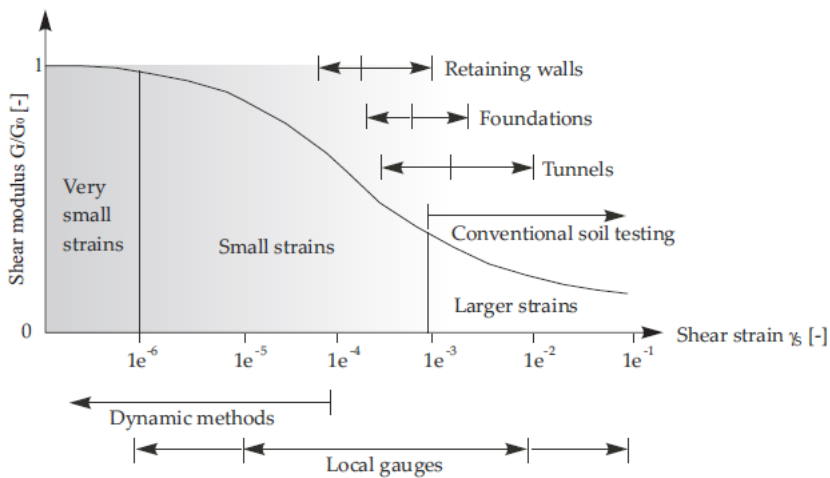


Figure 2.3 Stiffness-strain behaviour

### 2.2.1 CALIBRATION OF THE HS SMALL MODEL

In dynamic conditions, the soil is subjected to cyclic shear loading, showing not only a non linear but also a dissipative behaviour. The hysteretic loop generated during cyclic shear loading consists of a sequence of loading and unloading paths, because of the irreversible behaviour of the soil. In general, it has been observed that earthquakes induce a small strain level in the soil, that exhibits a high shear stiffness  $G_0$ , and that  $G$  decreases while the amount of dissipated energy increases by increasing the magnitude

of the shear strain  $\gamma$ .

To account for these aspects of material behaviour, the Hardening Soil model with small-strain stiffness (HS small model), based on the Hardening Soil model, is used in this example. The Hardening Soil model already accounts for the stress dependency of the stiffness according to a power law expressed by the  $m$  parameter. Compared to the Hardening Soil model, the HS small model is extended by introducing two additional parameters: the high stiffness at small strain level ( $G_0$ ) and the shear strain at which  $G$  has reduced to 70 % of the initial  $G_0$  ( $\gamma_{0.7}$ ).

The stress dependency is expressed by the following formula:

$$G_0 = G_0^{ref} \left( \frac{c \cdot \cos\varphi - \sigma'_3 \cdot \sin\varphi}{c \cdot \cos\varphi + p^{ref} \cdot \sin\varphi} \right)^m \quad (2.1)$$

where the initial shear stiffness  $G_0$  is a function of the effective stress, the strength parameters ( $c$  and  $\phi$ ) and the  $m$  parameter which depends on the soil type (it generally varies between 0.5 and 1, according to the soil type).

The typical hysteretic behaviour is shown in Figure 2.4. The initial tangent and secant stiffness of the initial loading curve coincide with the maximum shear stiffness  $G_0$ . By increasing the shear strain, the stiffness decays. When the load direction is inverted, the stiffness starts from the same  $G_0$  and decreases until the next load reversal. The stress-strain relationship is given by:

$$\tau = G_s \cdot \gamma \quad (2.2)$$

where  $G_s$  represents the secant shear stiffness.

The local hysteretic damping ratio is described by the following formula:

$$\xi = \frac{E_D}{4\pi E_S} \quad (2.3)$$

where  $E_D$  represents the dissipated energy, given by the area of the closed loop (yellow and green areas), and  $E_S$  is the energy accumulated at the maximum shear strain  $\gamma_C$  (green and blue areas). The damping ratio  $\xi$  applies until the material behaviour remains elastic and the shear modulus decreases with the strain.

To calibrate the parameters that need to be entered in PLAXIS it is suggested to refer to experimental data from site and laboratory tests performed in the chosen area. Considering the Eq. (2.1), it is possible to calibrate  $G^{ref}$  and  $m$  in order to have the best fitting. The decay of the shear modulus with strain is displayed in Figure 2.5. The green curve shows the ratio of the secant shear modulus over the initial shear stiffness  $G_s/G_0$  and the orange curve shows the ratio of the tangent shear modulus over the initial shear stiffness  $G_t/G_0$ , which can be calculated from Eq. (2.2) by taking the derivative with respect to the shear strain. The  $G_s/G_0$  curve is described in literature by Vucetic & Dorby (1991) according to different values of the plasticity index. In this example, the clay layer is characterized by a plasticity index PI of 50 % and the relative curve is displayed in blue. The value of  $\gamma_{0.7}$  has to be chosen in order to have the best fitting between the calculated  $G_s/G_0$  and the curve relative to the specific PI from Vucetic & Dorby (1991).

In the HS small model, the tangent shear modulus is bound by a lower limit,  $G_{ur}$ , to scale back to the original Hardening Soil model at higher strain levels.  $G_{ur}$  is related to  $E_{ur}$  and

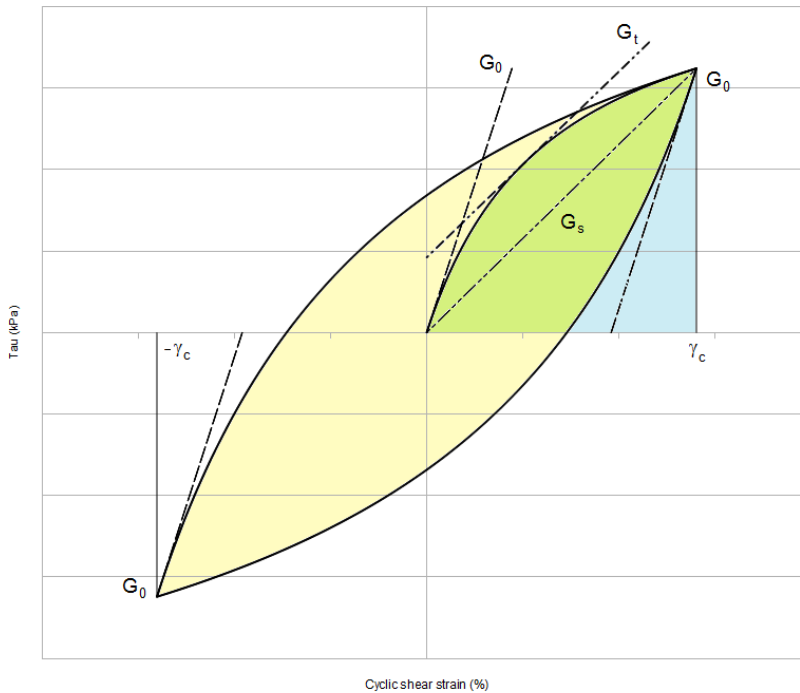


Figure 2.4 Hysteretic behaviour in the HSsmall model

$\nu_{ur}$  according to the following expression:

$$G_{ur} = \frac{E_{ur}}{2(1 + \nu_{ur})} \quad (2.4)$$

According to the values of  $E_{ur}^{ref}$  and  $\nu_{ur}^{ref}$  given in Table 2.1,  $G_{ur}^{ref}$  is equal to 4062.5  $kN/m^2$ , from Eq. (2.4). The ratio of  $G_0^{ref}$  over  $G_{ur}^{ref}$  generally varies between 2.5 and 10 times going from hard to soft soils. In this case, the ratio is about 6.8.

The shear strain that corresponds to the point in which the secant shear stiffness  $G_t$  reaches the value of  $G_{ur}$  represents the cut-off shear strain  $\gamma_{cut-off}$ , i.e. the limit above which the shear stiffness cannot decrease more than the reached  $G_{ur}$  value. The dashed curves on the right side of the cut-off shear strain represent the evolution of  $G_s/G_0$  and  $G_t/G_0$  with the shear strain without applying any limit.

The damping ratio evolves as a function of the shear strain and it increases for larger values of  $\gamma$ . In order to calibrate the value of  $\gamma_{0.7}$ , also the damping curve has to be taken into account. The Figure 2.6 represents the best fitting reached in this example, compared to the curve determined by Vucetic & Dorby (1991) for a clay characterized by PI equal to 50%. Also in this case, the cut-off shear strain represents the limit above which the damping ratio  $\xi$  cannot increase further.

A more detailed description and elaboration of the shear stiffness reduction curve and the damping curve can be found in the literature.

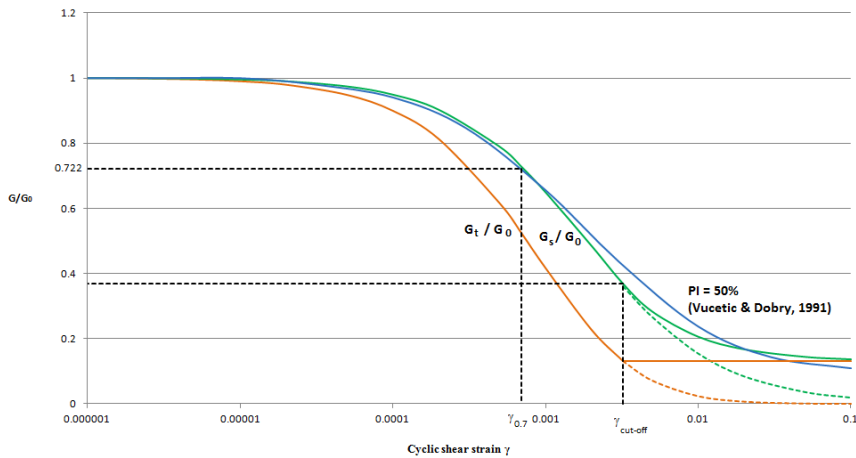


Figure 2.5 Modulus reduction curves

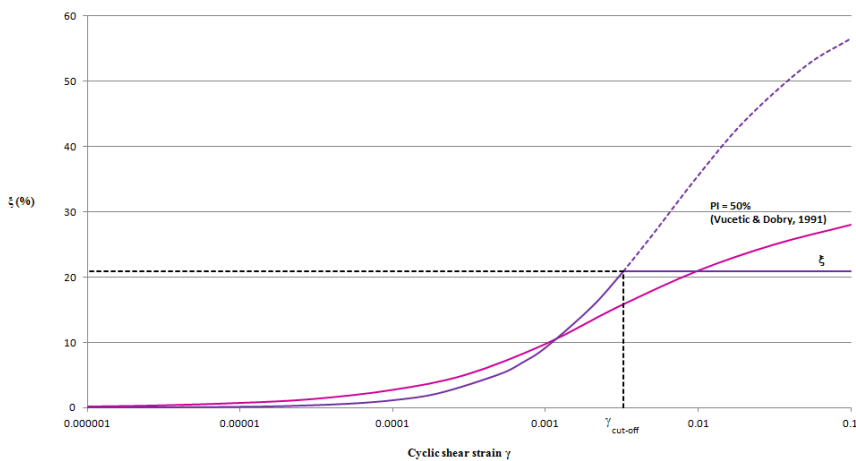


Figure 2.6 Damping curve

### 2.2.2 CALIBRATION OF THE RAYLEIGH DAMPING COEFFICIENTS

Hysteretic damping of the soil model can capture damping at strains larger than  $10^{-4}$  -  $10^{-2}$  %, depending on the values of material properties. Even at low deformation levels, the behaviour of the soil is irreversible. It is suggested to define the Rayleigh damping coefficients associated to a small damping ratio. According to the Rayleigh damping formulation, the damping matrix  $C$  is given by a portion of the mass matrix  $M$  and a portion of the stiffness matrix  $K$ , as a function of the Rayleigh coefficients  $\alpha$  and  $\beta$ :

$$[C] = \alpha[M] + \beta[K] \tag{2.5}$$

In the *General* tabsheet of the created material, by clicking on the *Rayleigh*  $\alpha$  box, the window expands on the right side and it is possible to calculate the Rayleigh coefficients by entering proper values for the damping ratios and the target frequencies (Figure 2.7).

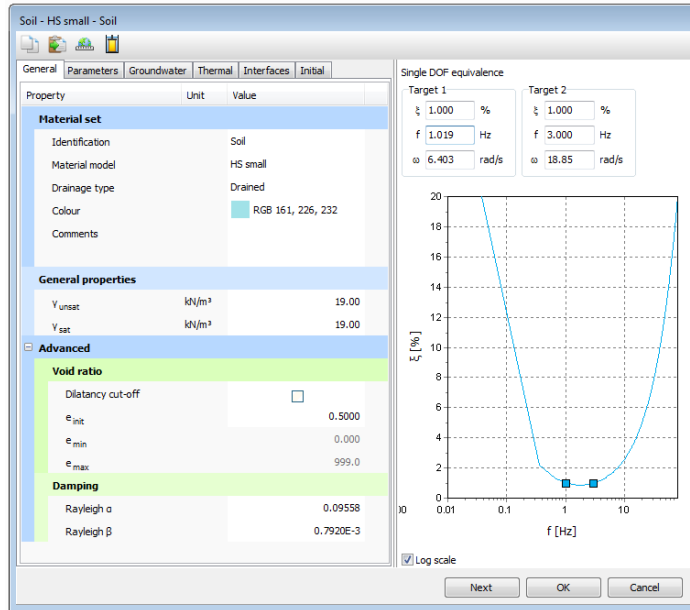


Figure 2.7 Rayleigh damping coefficients

To calibrate these two coefficients, it is necessary to define the target damping ratios  $\xi$  and the related frequencies. It is suggested to keep the same value of  $\xi$  for both Target 1 and 2, generally chosen between 0.5 and 2 %. Several strategies can be found in literature to select the appropriate frequencies. Hudson, Idriss & Beirkae (1994) proposed to set the first frequency equal to the fundamental frequency of the whole soil layer, and the second frequency as the closest odd number given by the ratio of the fundamental frequency of the input signal at the bedrock and the fundamental frequency of the whole soil layer. The fundamental frequency of the soil deposit is defined as the frequency at which the most significant amplification can be expected, and it corresponds to the first mode shape. According to this procedure, the frequency of Target 1 is given by:

$$f = \frac{v_s}{4H} \tag{2.6}$$

where  $v_s$  represents the shear wave velocity and  $H$  is the thickness of the soil layer. The value of  $v_s$  has been chosen as an average value over the whole depth. Based on the shear stiffness profile  $G$  and the unit weight of soil  $\gamma$ , it is possible to calculate the shear wave velocity profile with depth (Figure 2.8):

$$v_s = \sqrt{\frac{G}{\rho}} \tag{2.7}$$

where  $\rho$  is equal to the ratio of  $\gamma$  over the gravity acceleration  $g$ .

Considering that the average value of  $v_s$  is  $163.05 \text{ m/s}^2$  and the soil deposit is 40 m thick, the fundamental frequency is equal to 1.019 Hz. Considering the Fourier spectrum of the input signal at the bedrock, it can be found that the fundamental frequency of the signal is 2.46 Hz which gives a ratio  $2.46/1.019$  equal to 2.41 Hz. Therefore, the frequency of Target 2 is set equal to 3 Hz. Within the range of the chosen frequencies the

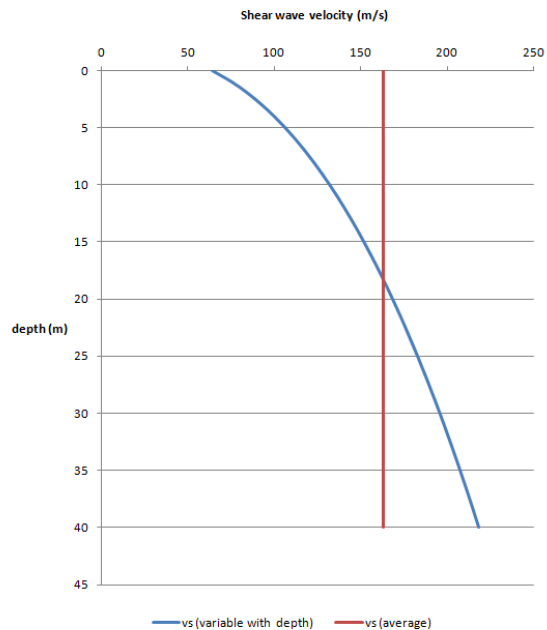


Figure 2.8 Shear wave velocity profile with depth and average shear wave velocity

damping is less than the target damping, whereas outside this range the signal is overdamped.

## 2.3 DEFINITION OF THE LOADING CONDITION

The ground motion input used in this example corresponds to the accelerogram recorded during Loma Prieta earthquake in 1989, characterized by a moment magnitude  $M_w$  equal to 6.9. The input signal is scaled at a peak horizontal acceleration of 0.1 g. The earthquake is assumed to be given at the bedrock and is modelled by imposing a prescribed displacement at the bottom boundary. To define the prescribed displacement:

- Click the *Structures* tab.
- Define a prescribed displacement at the bottom of the model, through (0.0 0.0) and (1.5 0.0).
- Set the x-component of the prescribed displacement to *Prescribed* and assign a value of 1.0 m. The y-component of the prescribed displacement is *Fixed*. The default distribution (*Uniform*) is valid (Figure 2.9).

To define the dynamic multipliers for the prescribed displacement:

- Expand the *Dynamic displacement* menu (*DynLineDisplacement\_1*)
- Click the *MultiplierX* drop down menu and click on the appearing plus button (+). The *Multipliers* window pops up and a new displacement multiplier is automatically added.
- In the *Name* section specify a name for the input signal.

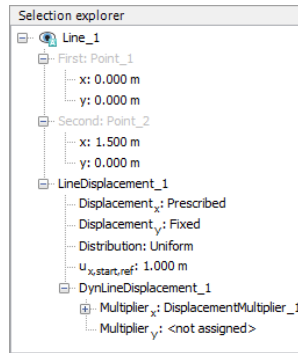


Figure 2.9 Definition of the prescribed displacement at the model bottom

- From the *Signal* drop-down menu select the *Table* option.
- The file containing the earthquake data is available in the PLAXIS knowledge base and you can save the file on your personal computer (<http://kb.plaxis.nl/search/site/loma0.1g>).
- In the *Multipliers* window click the *Open* button and load the file you have saved.
- Select the *Acceleration* option in the *Data type* drop-down menu.
- Select the *Drift correction* option and click *OK* to finalize the definition of the multiplier. The defined multiplier is displayed (Figure 2.10).

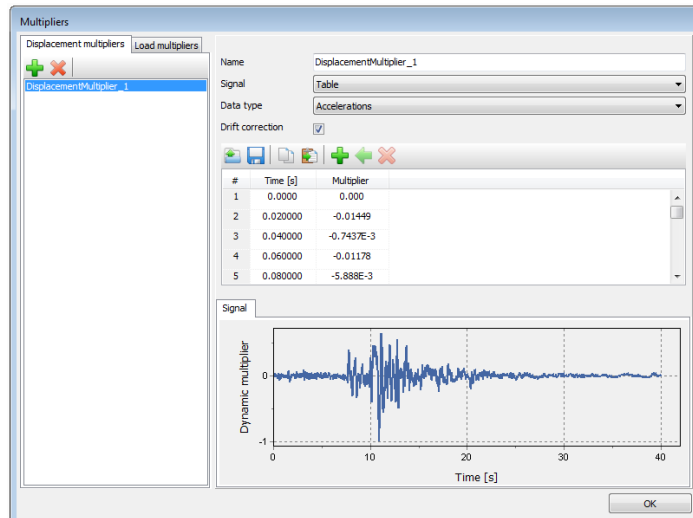


Figure 2.10 Dynamic multipliers window

The dynamic load is given in  $m/s^2$  and has a peak of  $1 m/s^2$ . By selecting the drift correction option, PLAXIS corrects the possible drift in the displacements (i.e. non null final displacement in the signal), caused by the integration of the accelerations and velocities. The correction is made by applying a low frequency motion from the beginning of the phase of the calculation and by correcting the acceleration accordingly. For a correct drift correction, the time interval of the phase and of the input signal should be the

same.



### 3 MESH GENERATION

- Proceed to the *Mesh* mode.
- Click *Generate mesh* and select *Fine* from the drop-down menu. Click *OK* (Figure 3.1).
- View the generated mesh. The resulting mesh is shown in Figure 3.2.
- Click the *Close tab* to close the *Output* program.

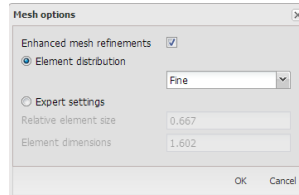


Figure 3.1 *Mesh options* window

The mesh generation is fully automatic and based on a robust triangulation procedure. The dimension of the triangular elements needs to be controlled and the mesh refinement allows to get a specific value for the average length of the element side. Kuhlmeier & Lysmer (1973) suggest to assume a size less than or equal to one-eighth of the wavelength associated with the maximum frequency component  $f_{max}$  of the input wave (i.e. the highest frequency component that contains appreciable energy):

$$AverageElementSize \leq \frac{\lambda}{8} = \frac{v_{s,min}}{8f_{max}} \quad (3.1)$$

where  $v_{s,min}$  is the lowest wave velocity. Based on the shear stiffness profile and the unit weight of soil, it is possible to calculate the shear wave velocity profile with depth (Figure 3.3). The lowest  $v_s$  is equal to 64.4 *m/s*. From the Fourier spectrum it can be found that the maximum frequency component is about 5 Hz, which leads to an average length of 1.6 m (Eq. (3.1)).

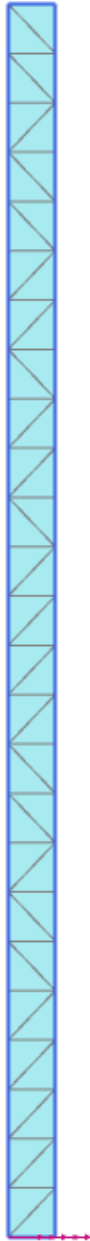


Figure 3.2 Generated mesh

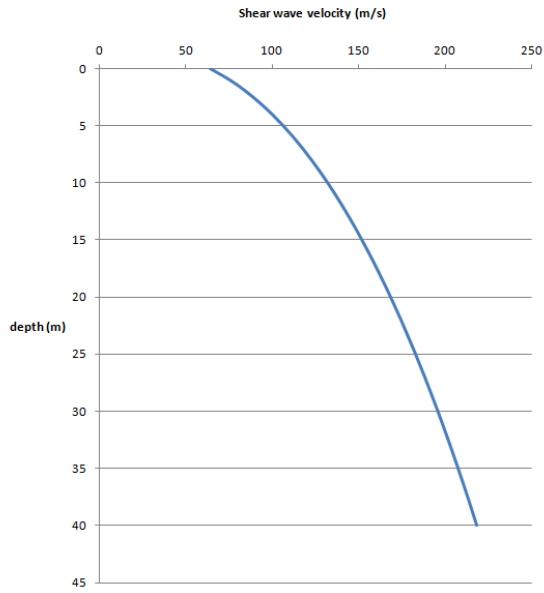


Figure 3.3 Shear wave velocity profile with depth



## 4 CALCULATION

The calculation process consists of the initial conditions phase and the free-field seismic analysis.

### **Initial phase**

- Click the *Staged construction* tab to proceed with the definition of the calculation phases.
- The initial phase consists of defining the initial geometry configuration and the initial stress state. The default settings of the initial phase will be used in this example.
- In the *Model explorer* expand the *Model conditions* subtree. The default setting is valid for this example A full fixity is generated at the base of the geometry, whereas roller supports are assigned to the vertical boundaries.
- In the *Staged construction* mode check that the prescribed displacement is inactive (Figure 4.1).

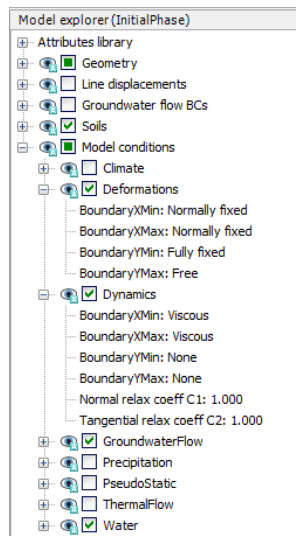


Figure 4.1 Initial phase

To be able to decide the calculation parameters for the dynamic phase, it is necessary to save the *Initial phase*. To start the calculation, click on the *Calculate* button.

Before starting the calculation, it is advised to select nodes and stress points at the depth of interest. The output of this analysis can be used, together with the output of other response analysis performed for the same site, to calculate the average response for the dynamic analysis of a building with shallow foundation. The foundation level is supposed to be at a depth of 3.6 m from the ground surface level. In order to show how the characteristics of the input signal are modified after propagating through the soil deposit and to calculate the amplification function, it is recommended to select a node at the bottom of the model:

- Select one node at the bottom of the soil column (0.75 0.0).

- Select one node and one stress point at (0.75 36.4).

### Phase 1

To apply the seismic signal at the geometry model:

- Add a new phase (*Phase\_1*).
- In the *Model explorer* expand the *Line Displacement* and the *LineDisplacement\_1* subtree.
- Activate *LineDisplacement\_1\_1* and *DynLineDisplacement\_1\_1*.
- Expand the *Model conditions* subtree.
- Switch off the option *Deformations*.
- Expand the *Dynamics* subtree. Set *BoundaryXMin* to tied degrees of freedom (Figure 4.2). The condition for the other vertical boundary *Xmax* is automatically set to tied degrees of freedom.

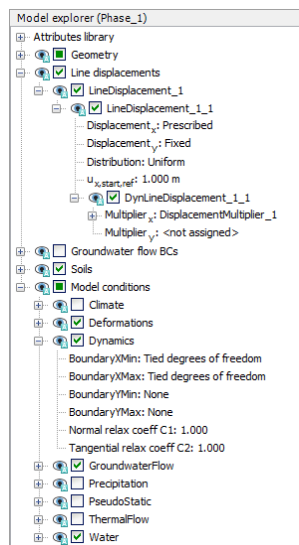


Figure 4.2 Boundary conditions for *Dynamic calculations* (Phase 1)

There are several possibilities to model the boundary conditions for a dynamic analysis. For the lateral boundaries, in the case of a one-dimensional wave propagation, the option *Tied degrees of freedom* allows to model a reduced geometry of the problem. It is sufficient to define a soil column with the horizontal dimension defined according to the average element size, while the nodes at the left and right model boundaries are connected to each other and are characterized by the same displacement. The *Tied degrees of freedom* boundary conditions can only be applied if the distribution of nodes along the two vertical model boundaries is identical, i.e the corresponding nodes at the left and right side should have the same y-coordinate. In PLAXIS, these boundary conditions only work on boundaries that are free to move. For this reason it is required to switch off the default fixities. As for the base of the model, a fully reflective boundary is simulated through a fixed base and the applied input signal is supposed to be recorded at the depth of the bedrock. The dynamic boundary condition at the base of the model

( $Y_{min}$ ) is set to *None* and the line prescribed displacement is activated.

To define the calculation parameters:

- In the *Phases* window select the *Dynamic* option as *Calculation type*.
- Set the *Dynamic time interval* parameter to 40 sec (Figure 4.3).
- Deselect the *Use default iter parameters* box to be able to enter manually the maximum number of steps.
- Enter a value of 2000 for *Max steps*.
- Select *Manual* option for the *Time step determination*, *The number of substeps* box becomes editable. In the right panel, click the button *Retrieve* below the table. The necessary number of substeps is then calculated. Click *Apply* to use the calculated value.
- Keep the default settings for the other numerical control parameters.
- Keep the default values of *Alpha-Newmark* and *Beta-Newmark* (0.25 and 0.5, respectively) and *Mass matrix*.

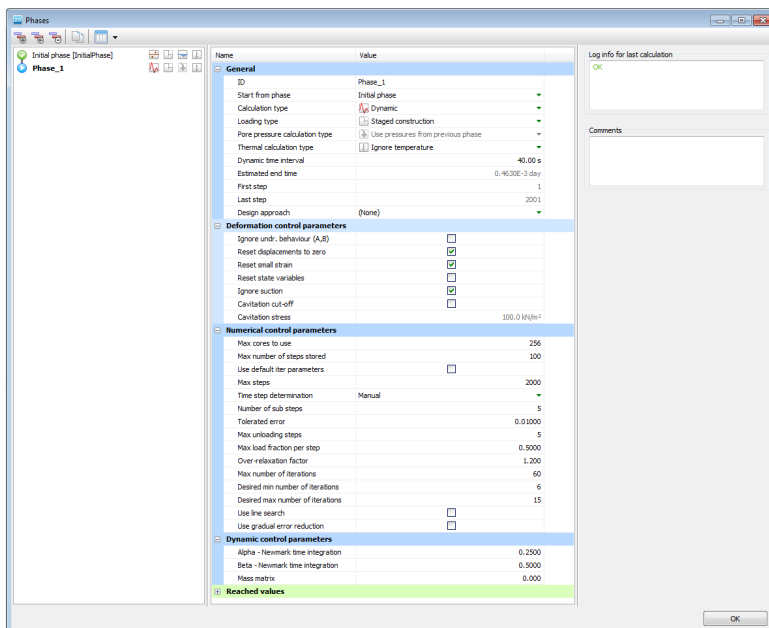


Figure 4.3 *Phases* window for *Dynamic calculations*

The automatic procedure implemented in PLAXIS ensures that a wave does not cross more than one element per time step. The conditions to be verified are two: at first, the critical time step is estimated according to the element size and the material stiffness, then the time step is adjusted based on the number of data points specified as dynamic multipliers.

The time step  $\delta t$  used in a *Dynamic calculation* is given by:

$$\delta t = \frac{\Delta t}{m \cdot n} \quad (4.1)$$

where  $\Delta t$  is the *Dynamic time interval* parameter, i.e. the duration of the earthquake,  $m$  is the *Max steps number* and  $n$  is the *Number of sub steps*. PLAXIS automatically calculates the proper number of sub steps  $m$ , for a given number of steps  $n$ .

It is advised to set the number of maximum steps equal to the number of multipliers that define the input signal. The upper limit is given by 10000 number of steps. To check if the used signal is the same as the one given in input, it is possible to create a chart in the *Output* program selecting *Dynamic time* on the X-axis and  $a_x(g)$  on the Y-axis at the bottom level. The signal given in this chart should be the same as the input signal. By modifying the calculated values and using a time step that is larger than the specified limits, the data are interpolated and this may lead to inaccurate results.

**Hint:** If the dynamic displacement or load is not activated and the previous phase has not been performed yet, the iteration parameters related to the dynamic multipliers cannot be correctly calculated.

To start the calculation:

- Click *Calculate*.
- Save the project after the calculation has finished.



## 5 RESULTS

Click *View calculation results*. The deformed mesh at the end of the analysis is shown in Figure 5.1.

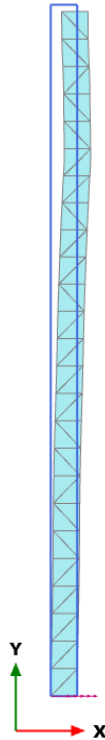


Figure 5.1 Deformed mesh at the end of the analysis.

From the output menu, it is possible to show the distribution of some significant parameters such as the shear deformation  $\gamma_{xy}$  and the ratio of the tangent over the unloading-reloading shear stiffness modulus  $G_t/G_{ur}$ .

- From the *Deformations* menu select  $\gamma_{xy}$  under the *Total cartesian strains* menu (Figure 5.2).
- The distribution of the shear deformation at the end of the analysis is shown.

The maximum value does not represent the maximum shear deformation reached during the analysis but its value at the end.

- From the *Stresses* menu select  $G_t/G_{ur}$  under the *State parameters* menu (Figure 5.3).
- The decay of the tangent shear stiffness in the soil deposit, due to the shear deformations reached during the earthquake, is shown.

The minimum value of  $G_t/G_{ur}$  at the end of the analysis is 1.354 which means that the tangent stiffness modulus has almost reached the lower limit imposed by  $G_{ur}$ . The shear strain associated to this ratio is equal to 0.002512.



Figure 5.2 Shear strain deformation  $\gamma_{xy}$

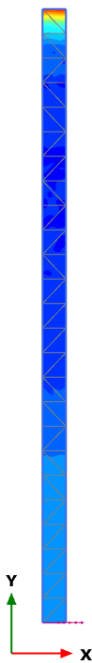


Figure 5.3 Ratio of the tangent over the unloading-reloading shear stiffness modulus  $G_t/G_{ur}$

## 5.1 CURVES

The output of a dynamic calculation can be plotted in charts, both in time and in frequency domain. Click on the *Curve manager* icon to start.

### 5.1.1 ACCELEROGRAM

The accelerogram represents the time history plot of an earthquake in terms of acceleration. In the *Curves manager* section, create a new chart (Figure 5.4):

- On the X-axis select *Project* and *Dynamic time* under the *Time* subtree.
- For the Y-axis, select the point B (0.75 36.4). Under the *Deformations* subtree, expand the *Acceleration* field and select  $a_x('g')$ , as generally the accelerograms are plotted in a normalized plane, i.e. the acceleration is divided by the gravity acceleration  $g$ .
- Click *OK* to generate the curve.

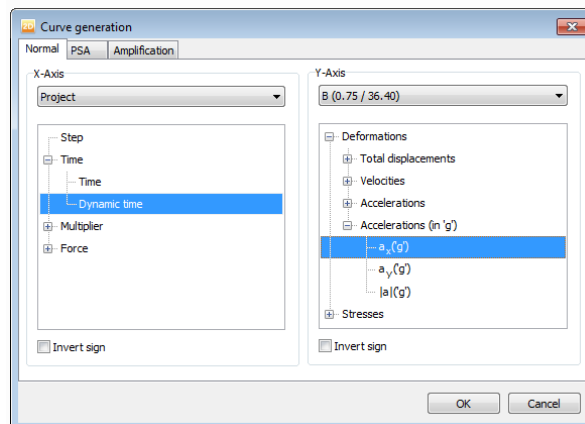


Figure 5.4 Generate curve

The peak acceleration at the selected depth is equal to  $-0.186 g$  and represents the maximum amplitude of the acceleration at the depth of interest (Figure 5.5). The signal has been filtered by the clay layer, since its peak acceleration has increased from  $-0.1 g$  to  $-0.186 g$ . The peak horizontal acceleration is closely related to the dynamic forces induced on a structure by the strong motion but it is not sufficient to describe the earthquake characteristics.

### 5.1.2 FAST FOURIER TRANSFORM

The frequency content gives information about how the amplitude of a ground motion is distributed among different frequencies. This information is necessary since the dynamic response of a structure depends on both the amplitude of the applied loads and the frequency range in which the energy content of the seismic signal is concentrated. It is possible to transform the generated accelerogram in the frequency domain, obtaining the Fast Fourier Transform, i.e. the power spectrum:

- Select *Settings* under the *Format* menu; the *Settings* window opens.

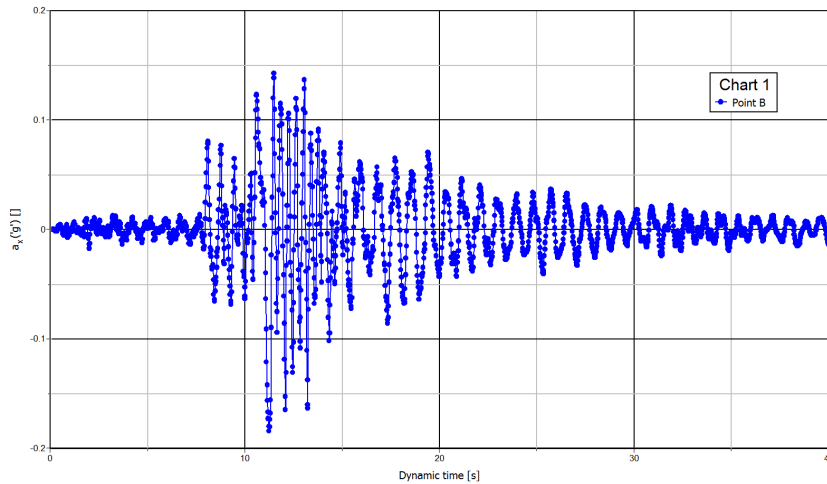


Figure 5.5 Accelerograms at 3.6 m from the top level

- Select the *Chart* tabsheet and mark the *Use frequency representation (spectrum)* and *Standard frequency (Hz)* options in the *Dynamics* box (Figure 5.6).
- Click *OK* to generate the curve.

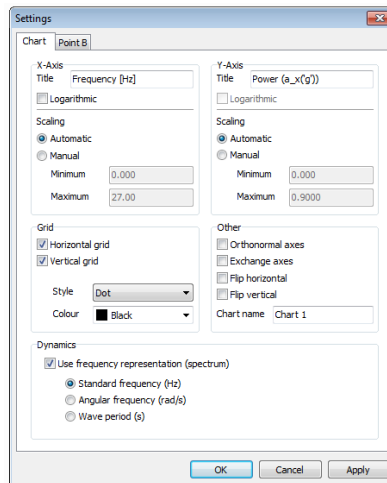


Figure 5.6 Generation of Fast Fourier Transform

The frequency that corresponds to the maximum amplitude at the foundation level is equal to 1.12 Hz. Part of the energy content is also concentrated between 2.3 and 2.8 Hz (Figure 5.7). Due to the filter effect of the soil deposit, the energy content of the input earthquake has been modified as shown in Figure 5.8 where the FFT curve at the bottom of the model (bedrock) is added to the chart.

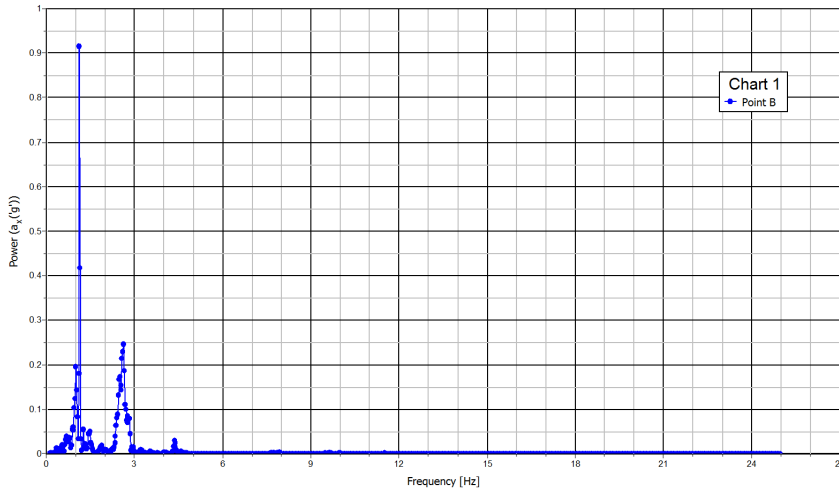


Figure 5.7 FFT at the foundation level

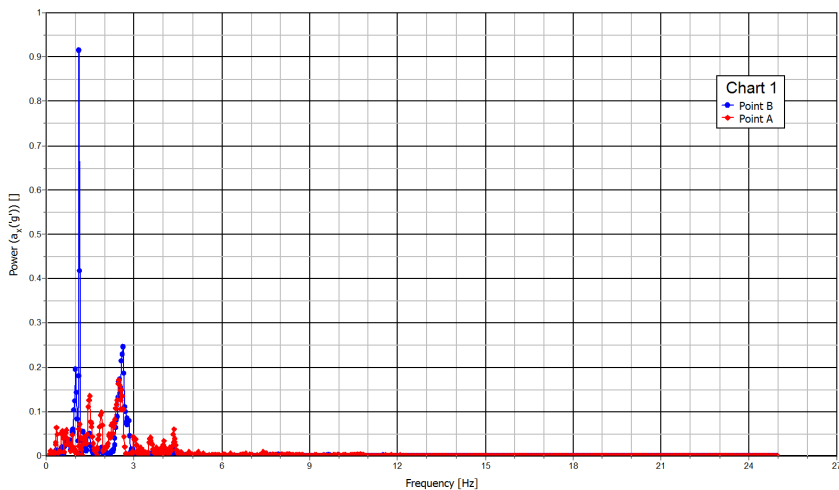


Figure 5.8 Comparison of the FFT at the bottom and at the foundation level

### 5.1.3 RESPONSE SPECTRUM

The maximum resulting forces acting on a structure as a consequence of an earthquake can be calculated when the structure, idealized as a single degree of freedom system, reaches the maximum deformation. A detailed description of the response spectrum can be found in the *Reference manual*.

In PLAXIS the response spectrum is calculated as the response of SDOF systems in terms of accelerations, characterized by different stiffness  $k$  but same damping ratio  $\zeta$  and subjected to the same earthquake. The information about the structure stiffness  $k$  is contained in the natural period  $T$ , reported on the X-axis of the response spectrum chart,

as shown in the following equation:

$$T = 2\pi\sqrt{\frac{m}{k}} \quad (5.1)$$

To generate the response spectrum:

- Click on the *Curve manager* icon and click *New*.
- Select the *PSA* tab from the *Curve generation* window. Select the node B (0.75 36.4) and  $a_x$ . Keep the default damping ratio of 5 %, typical of structures in reinforced concrete (Figure 5.9).
- Click *OK* to generate the curve.

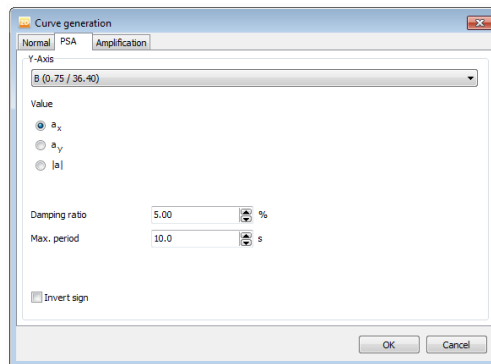


Figure 5.9 Generate PSA

The value that can be read on the Y-axis in correspondence of the natural period of vibration of the overlying structure allows to calculate the maximum shear stress at the base of the structure (Figure 5.10):

$$F_{max} = m \cdot PSA \quad (5.2)$$

The peak value in the PSA spectrum occurs at a period of 0.4 s, which means that, if the natural period of the overlying structure is 0.4 s, the system is in resonance.

To generate the relative displacement response spectrum:

- Selecting *Settings* under the *Format* menu.
- Select the *Chart* tabsheet in the *Settings* window and the *Relative displacement response spectrum* option.

The generated chart (Figure 5.11) gives information about the maximum displacement that can occur in the structure. The product of the maximum displacement and the stiffness  $k$  represents the maximum equivalent static force, i.e. the static force that causes the same maximum deformation induced by the earthquake.

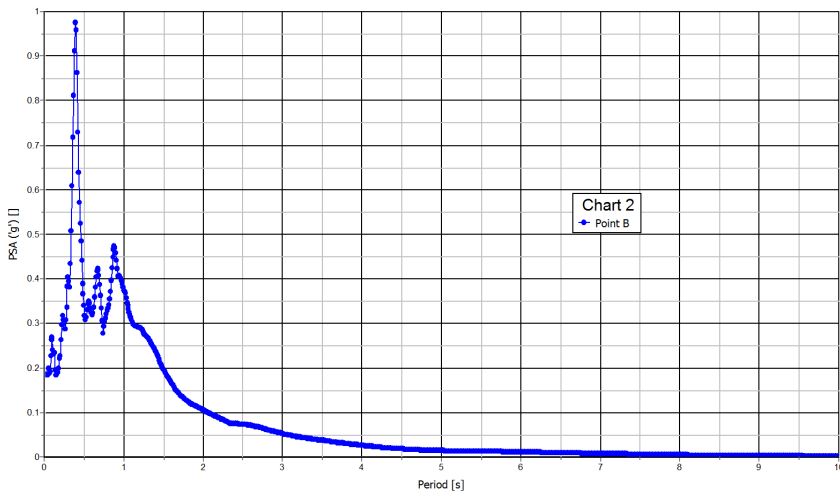


Figure 5.10 PSA

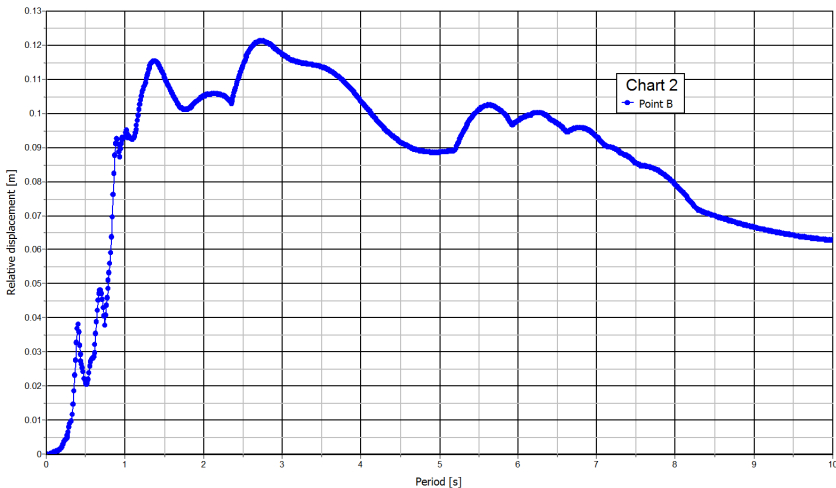


Figure 5.11 Relative displacement response spectrum





## 6 COMPARISON OF THE RESULTS WITH EERA

The results obtained in this example are now compared with the results obtained by a 1D equivalent linear visco-elastic analysis in the frequency domain.

### 6.1 INTRODUCTION TO EERA

In 1998, the computer program EERA (Equivalent-linear Earthquake Response Analysis) (Bardet, Ichii & Lin (2000)) was developed in FORTRAN 90 starting from the same basic concepts as SHAKE (Schnabel, Lysmer & Seed (1972)). EERA allows for one-dimensional wave propagation analysis in a soil deposit and is based on a frequency domain formulation. When subjected to a cyclic load, the soil usually exhibits a non linear hysteretic stress-strain behaviour that can be approximated with equivalent linear soil properties, such as the secant shear modulus  $G$  and the damping ratio  $\xi$  which represent the inclination and the width of the hysteretic loop, respectively. According to the linear approach,  $G$  and  $\xi$  are constant for each soil layer and, starting from the initial estimated values, they are updated in subsequent iterations to be consistent with the level of strain induced in each layer. The iteration procedure for equivalent linear approach can be summarized as follows for each layer:

- Initialize the values of  $G^{(i)}$  and  $\xi^{(i)}$  at their small strain values.
- Compute the ground response, and get the amplitudes of maximum shear strain  $\gamma_{max}$  from the time histories of shear strain in each layer.
- Determine the effective shear strain  $\gamma_{eff}^{(i)}$  as  $R_\gamma \cdot \gamma_{max}^{(i)}$ , where  $R_\gamma$  is the ratio of the effective shear strain to maximum shear strain, which depends on the earthquake magnitude  $M$  and is the same for all layers:

$$R_\gamma = \frac{M - 1}{10} \quad (6.1)$$

- Calculate the new equivalent linear values  $G^{(i+1)}$  and  $\xi^{(i+1)}$  corresponding to the effective shear strain  $\gamma_{eff}^{(i)}$ .
- Repeat the iteration until the differences between the computed values of shear modulus and damping ratio in two successive iterations is less than a specified error in all layers.

The input and output information in EERA are fully integrated with the spreadsheet program Microsoft Excel and are organized in a minimum of nine worksheets: some of them can be duplicated and modified, for example to obtain output at several sub-layers or to define additional material curves.

The input worksheets are:

- *Earthquake*: the input earthquake is defined by the time interval of its data points, the desired maximum acceleration (in 'g') and the maximum frequency cut-off which allow to scale and filter the input motion, the number of data points to be used for the Fast Fourier Transform algorithm.
- *Profile*: the soil is subdivided in a number of homogenous layers. Each layer is characterized by its thickness, the initial shear stiffness modulus or the shear wave

velocity, the initial damping ratio (only if no material curve is defined), the unit weight of soil, the number of material type. Each material type is defined in a separate worksheet called *Mat i*. The location and type of earthquake motion is defined by selecting at the corresponding depth either the option *Outcrop* for an outcropping rock motion, or *Inside* for a non outcropping motion. The depth of the water table can be specified in order to calculate the vertical effective stresses.

- *Mat i*: several material stress-strain and damping-strain curves can be defined by assigning the values of shear strain and the corresponding stiffness and critical damping ratio.
- *Iteration*: the number of iterations has to be specified in order to achieve a satisfactory convergence. The ratio of equivalent uniform strain is entered to account for the effects of the earthquake magnitude. The type of linear equivalent model is selected.

The results are organized in the following worksheets that can be duplicated to get the information at different depths:

- *Acceleration*: it defines the time history of acceleration, relative velocity and relative displacement at the top of the selected sublayer.
- *Strain*: defines the time history of stress, strain and dissipated energy, and stress-strain loops in the middle point of the selected sublayer.
- *Ampli*: defines the amplification factor between two selected sublayers.
- *Fourier*: defines the Fourier spectrum at the top of the selected sublayer.
- *Spectra*: defines response spectra in terms of displacement, velocity and acceleration at the top of the selected sublayer.

## 6.2 INPUT

The input used in EERA is shown in the following figures. The input signal (Figure 6.1) is exactly the same as the one used for the model in PLAXIS and is assumed to be applied at the bedrock. The bedrock is simulated by adding one layer below the soil deposit with considerably higher stiffness properties and selecting the option *Inside* to indicate the type and location of the input motion. The soil layer is discretized in 17 sublayers (Figure 6.2), characterized by the same unit weight of soil but different shear wave velocity  $v_s$  to simulate its dependency from the initial stress state (Figure 6.3). The values of  $G$  has been chosen for each sublayer in order to approximate the shear modulus variation with depth given by PLAXIS (Figure 6.4). The properties are constant within the sublayer. The degradation of the shear modulus and the evolution of the damping ratio with the reached level of strain follow the curves in the tabsheet *Mat 1*, which corresponds to a clay with Plasticity Index about 50 % (Figure 6.5). No degradation of the material properties of the bedrock is considered, i.e. the selected material type is 0. The input signal refers to an earthquake of magnitude of 6.9, which is taken into account in the *Ratio of effective and maximum shear strain* in the *Iteration* tabsheet (Figure 6.6) and it is equal to 0.59 according to Eq. (6.1). The number of iterations has been chosen in order to get a very small error when achieving convergence.

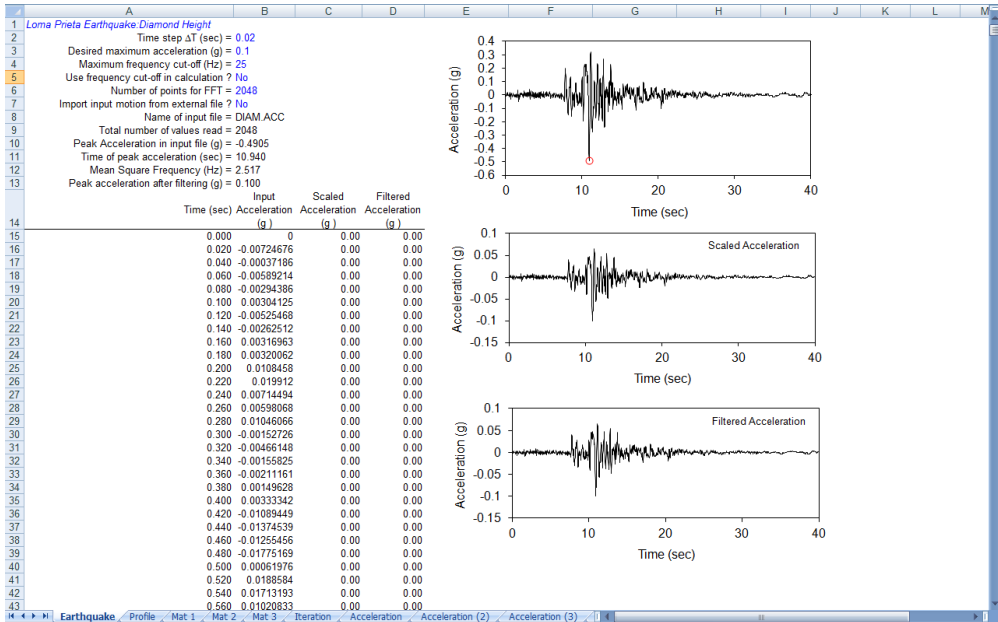


Figure 6.1 Earthquake tabsheet, EERA

Layer Number	Soil Material Type	Number of sublayers in layer	Thickness of layer (m)	Maximum shear modulus $G_{max}$ (MPa)	Initial critical damping ratio (%)	Total unit weight (kN/m <sup>3</sup> )	Shear wave velocity (m/sec)	Location and type of earthquake input motion	Location of water table	Depth at middle of layer (m)	Vertical effective stress (kPa)
6	Surface	1	1.0	9.58	19.00	70.32128457				0.5	9.50
7		2	1.0	12.56	19.00	80.5433876				1.5	28.50
8		3	1.6	16.17	19.00	91.3832455				2.8	53.20
9		4	1.4	20.14	19.00	101.96248				4.3	81.70
10		5	1.5	23.77	19.00	110.793513				5.8	109.25
11		6	2.0	27.98	19.00	120.1940407				7.5	142.50
12		7	2.0	32.61	19.00	129.764866				9.5	180.50
13		8	2.0	37.09	19.00	138.3769825				11.5	218.50
14		9	2.0	41.43	19.00	146.2507509				13.5	256.50
15		10	2.0	45.66	19.00	153.5343357				15.5	294.50
16		11	3.0	50.80	19.00	161.9459638				18.0	342.00
17		12	3.0	56.81	19.00	171.2700196				21.0	399.00
18		13	3.0	62.67	19.00	179.8878986				24.0	456.00
19		14	3.0	68.40	19.00	187.9265592				27.0	513.00
20		15	3.0	74.01	19.00	195.4795475				30.0	570.00
21		16	4.0	80.41	19.00	203.760284				33.5	636.50
22		17	4.5	88.03	19.00	213.1897404				37.8	717.25
23	Bedrock	18	0	3336.48	1	22.02	1219.2	Inside		40.0	760.00

Figure 6.2 Profile tabsheet, EERA

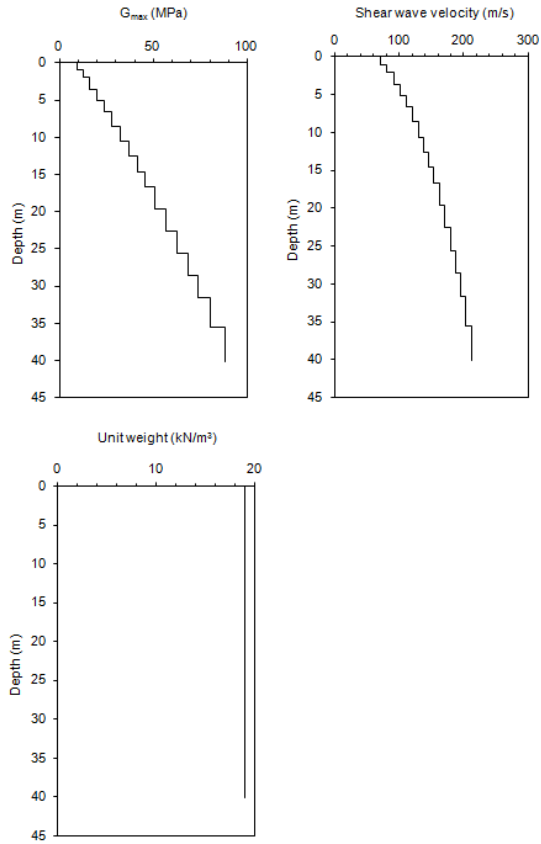


Figure 6.3 Variation of  $G$ ,  $v_s$  and Unit weight of soil with depth. *Profile* tabsheet, EERA

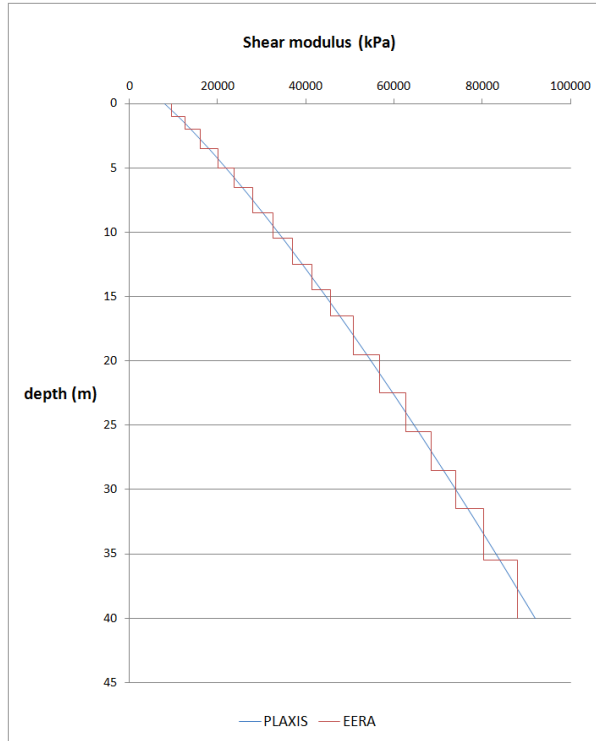


Figure 6.4 Comparison of the shear modulus profile with depth in PLAXIS and EERA

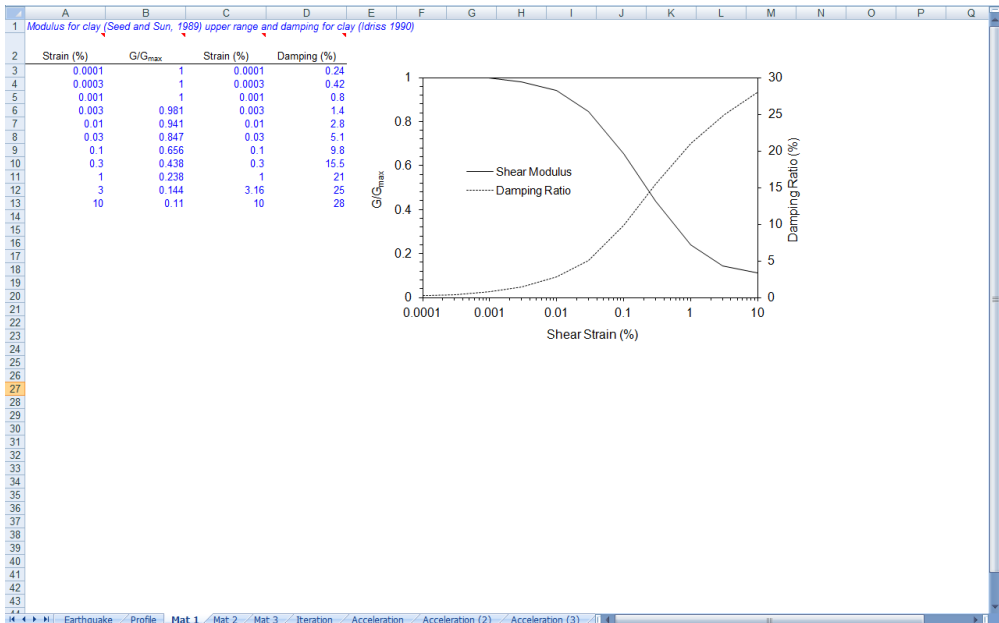


Figure 6.5 Mat 1 tabsheet, EERA

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Number of iterations: 14														
2	Ratio of effective and maximum shear strain: 0.59														
3	Type of shear modulus = Shake91														
4	Convergence achieved (%) = 0.00031														
Iteration Number	Sublayer Number	Type	Depth (m)	Maximum Strain (%)	Time of Maximum Strain (sec)	Shear Modulus	G/G <sub>max</sub>	Damping (%)	Convergence on Shear Modulus (%)	Convergence on Damping (%)	Maximum stress (kPa)	Depth at top of sublayer (m)	Maximum acceleration (g)		
5	1	1	1	0.5	0.029062	19.38	9.577631	1	0.24	0	0	2.783411			
6	1	2	1	1.5	0.063527	21.06	12.56448	1	0.24	0	0	7.98182			
7	1	3	1	2.8	0.088182	21.08	16.17401	1	0.24	0	0	14.26265			
8	1	4	1	4.35	0.104303	21.08	20.13564	1	0.24	0	0	21.0021			
9	1	5	1	5.85	0.111542	21.1	23.7746	1	0.24	0	0	26.51866			
10	1	6	1	7.6	0.116804	23.32	27.98018	1	0.24	0	0	32.68207			
11	1	7	1	9.6	0.118399	23.32	32.6136	1	0.24	0	0	38.61416			
12	1	8	1	11.6	0.118645	21.56	37.0962	1	0.24	0	0	44.0009			
13	1	9	1	13.6	0.117271	21.58	41.42674	1	0.24	0	0	48.58147			
14	1	10	1	15.6	0.114765	21.56	45.65576	1	0.24	0	0	52.39668			
15	1	11	1	18.1	0.111539	22.02	50.79545	1	0.24	0	0	56.65688			
16	1	12	1	21.1	0.10729	22.02	56.81294	1	0.24	0	0	60.9548			
17	1	13	1	24.1	0.104468	18.94	62.67415	1	0.24	0	0	65.47433			
18	1	14	1	27.1	0.104659	18.92	68.40076	1	0.24	0	0	71.58778			
19	1	15	1	30.1	0.10297	20.7	74.00946	1	0.24	0	0	76.20725			
20	1	16	1	33.6	0.098217	22.44	80.41251	1	0.24	0	0	78.97895			
21	1	17	1	37.95	0.093749	22.44	88.02726	1	0.24	0	0	82.52455			
22	1	18	0	40.1	0.002491	22.44	3336.482	1	1	0	0	83.11804			
23															
24															
25	2	1	1	0.5	0.032146	11.24	8.570684	0.894865	3.928841	10.51352406	1537.01709	2.755138			
26	2	2	1	1.5	0.07872	11.24	10.19835	0.811681	5.969102	18.83188057	2387.125977	8.130173			
27	2	3	1	2.8	0.119239	11.24	12.28668	0.759655	7.249316	24.03445053	2920.54834	14.65056			
28	2	4	1	4.35	0.148595	11.26	14.75984	0.733021	7.904727	26.69793129	3193.636475	21.93242			
29	2	5	1	5.85	0.163192	11.26	17.1742	0.722376	8.166667	27.7624073	3302.778076	28.02698			
30	2	6	1	7.6	0.168713	11.26	20.00758	0.715063	8.346627	28.49374199	3377.761719	33.75545			
31	2	7	1	9.6	0.16612	11.24	23.25061	0.712912	8.39956	28.70884895	3399.81665	38.62396			
32	2	8	1	11.6	0.160496	11.24	26.42697	0.712582	8.407661	28.74178979	3403.192393	42.41419			
33	2	9	1	13.6	0.151505	11.22	29.59652	0.71443	8.362185	28.55096106	3384.243652	44.84019			
34	2	10	1	15.6	0.143575	11.22	32.77432	0.717857	8.277855	28.21426201	3349.106445	47.05574			
35	2	11	1	18.1	0.134473	11.2	36.69361	0.72238	8.166572	27.76202202	3302.738281	49.34311			

Figure 6.6 Iteration tabsheet, EERA

### 6.3 RESULTS

The results at a depth of 3.6 m from the surface level are compared in term of time history horizontal acceleration and horizontal velocity, PSA and Relative displacement spectra. The results obtained in PLAXIS are in good agreement with the output of EERA. Small differences may be due to the different hypothesis at the origin of the calculations: while EERA considers a viscoelastic linear soil, PLAXIS accounts for plasticity in the HS small model. One additional reason may be related to the boundary condition that has been set at the bottom of the model: when a fully reflective boundary is modelled, the downward propagating waves are reflected back into the model, leading to large amplitude periodic vibrations.

In Figure 6.7 the comparison of the accelerograms generated by PLAXIS and EERA is shown. Even though the maximum acceleration in PLAXIS is lower than the peak acceleration in EERA (-0.186 g instead of -0.230 g), the general distribution with time is consistent and the time of the maximum acceleration is 11.24 s in PLAXIS and 11.26 s in EERA.

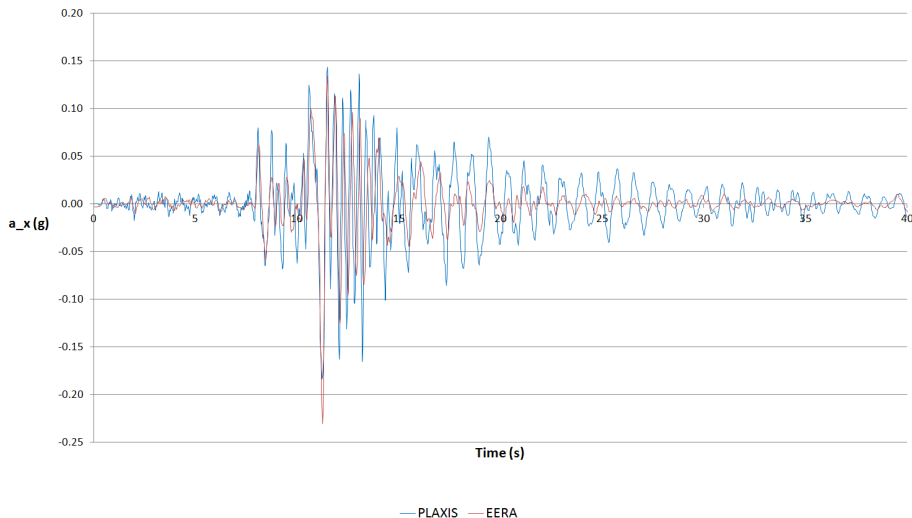


Figure 6.7 Accelerogram at 3.6 m from the surface level. Comparison PLAXIS - EERA

The time history velocity plot (Figure 6.8) shows a good agreement of the results obtained in PLAXIS and EERA, with a peak velocity of 0.2533 m/s in the horizontal direction in PLAXIS slightly larger than the peak velocity of 0.2490 m/s in EERA.

In the PSA spectrum (Figure 6.9) it can be seen that the predominant period is the same and it is equal to 0.4 s. The maximum spectral acceleration is slightly overestimated in PLAXIS, giving a value of 1.03 g against the maximum PSA in EERA equal to 0.7378 g.

The maximum relative displacement (Figure 6.10) both in EERA and in PLAXIS is equal to 0.122 m.

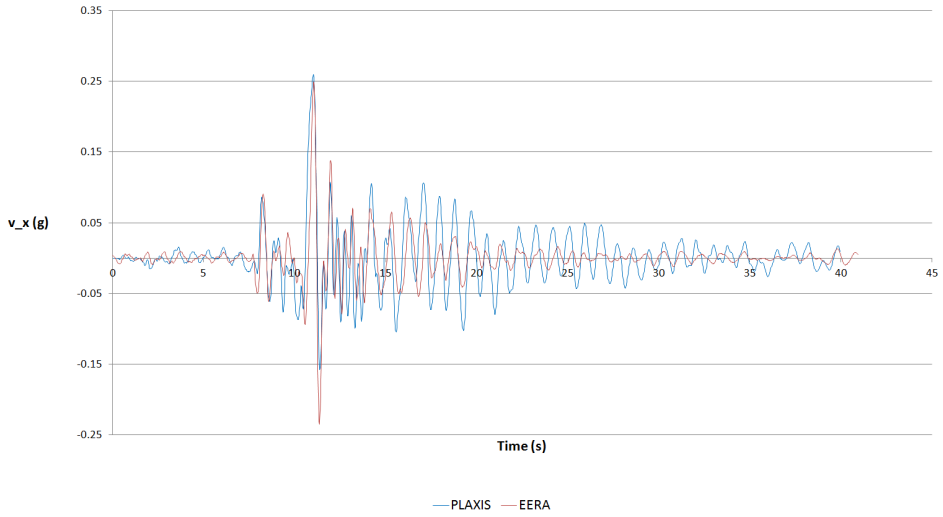


Figure 6.8 Time history velocity at 3.6 m from the surface level. Comparison PLAXIS - EERA.

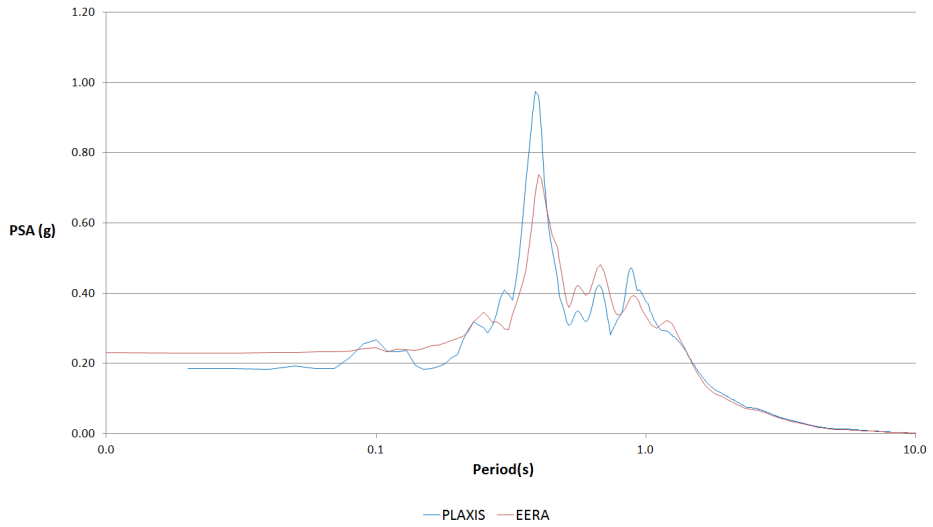


Figure 6.9 PSA spectrum at 3.6 m from the surface level. Comparison PLAXIS - EERA.



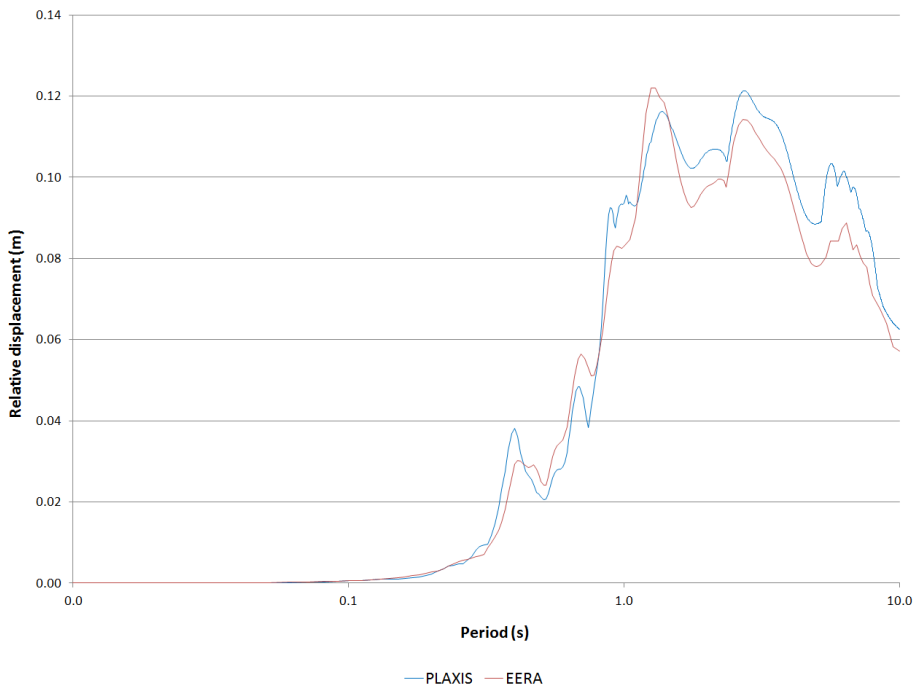


Figure 6.10 Relative displacement response spectrum at 3.6 m from the surface level. Comparison PLAXIS - EERA.



## 7 CONCLUSIONS

This example presents the results of a one dimensional site response analysis performed with PLAXIS 2D , aimed at modelling the non-linear dissipative behaviour of soils subjected to an earthquake loading. In this respect, an important role is played by the choice of the constitutive model for the soil. The results of the fully dynamic analysis performed with PLAXIS 2D are compared to an equivalent-linear site response analysis performed with EERA, where the secant shear modulus and the damping ratio are updated in subsequent iterations to be consistent with the level of strain induced in each layer. In PLAXIS , the Hardening Soil model with small strain stiffness has been chosen showing that, for earthquakes characterized by moderate peak accelerations, it is capable to capture the soil behaviour during earthquake shaking. The results from both approaches are compared showing a good agreement.

Some limitations have been experienced when the HS small model is used in combination with strong motions, even though it is not possible to simply identify a threshold peak acceleration for the input motion since more than one characteristics have an influence, such as the frequency content of the input motion, the mechanical properties of the soil, the boundary conditions, etc. In this case, a possible alternative may be to use the user-defined soil model known as Generalized Hardening Soil model.



## 8 REFERENCES

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